Modified voltage stability index for radial distribution system

Asha Gaikwad1, Rakesh Ranjan2 and L.D. Arya3
1Dept. of Electrical Engg., G.H. Raisoni Institute of Engineering and Technology, Pune-412207, India
2International Institute of Technology and Business, Sonipat, Gurgaon, India
3Dept. of Electrical Engg., Shri Govindram Seksaria Institute of Technology and Science, Indore-452003 (M.P.), India
asha_asmi@yahoo.com; +91 9881593553

Abstract
This study presents a new voltage stability index for identifying the node which is most sensitive to voltage collapse. Using this voltage stability index, it is possible to compute the stability index value at every node and the node at which the value of the voltage stability index is minimum is most sensitive to voltage collapse. The effectiveness of the proposed technique has been demonstrated through a 32-node and 69-node Radial Distribution System (RDS). It is shown that with the proposed voltage stability index for 69-node system, the node which is more sensitive to voltage collapse is node-65 which is the same as obtained by the voltage stability index developed by Chakravorty and Das (2001).

Keywords: Voltage stability index, voltage collapse, radial distribution system, 69-node system.

Introduction
Normally, distribution systems comprise of loads like industrial, commercial, domestic, lighting and others. Each of these loads is at its maximum at different times of the day and this may cause feeder overloading which may result in voltage collapse. Low voltage causes loss of revenue, inefficient lighting and possible burning out of motors. Milanovic and Yan Zhang (2010) have shown that the overall financial losses in the network due to voltage sags can be reduced by optimal placement of three FACTS devices namely, static VA compensator (SVC), static compensator (STATCOM) and dynamic voltage restorer (DVR) using a Niching genetic algorithm (NGA). Zhu et al. (2010) discussed the application of a coordinated VAr compensator (SVC) as an additional control in reactive power (VAR) optimization problem and analyzed the impact on system loss minimization and voltage improvement.

Static preventive-corrective optimizations cannot account for the speed at which post-contingency corrective actions are applied. It may happen that the post-contingency system evolution voltage is unstable because the statically computed corrective controls are not acting fast enough even when applied at the maximum allowed rate. To handle such situations in the case of long-term voltage stability, an approach coupling static optimization with time-domain simulation has been proposed by Capitanescu et al. (2009). A new approach for enhancement of voltage stability by network reconfiguration was proposed by Kashem et al. (2000). A method for the online testing of power system is proposed by Kesse and Glavitsch (1986) which aimed at the detection of voltage instabilities. A fast method to determine the voltage stability limit of power system was proposed by Haque (1995).

Three simple voltage stability indices are proposed in to determine the voltage instability of power system (Sinha and Hazarika, 2000). A method using parallel self-organizing hierarchical neural network (PSSHNN) is proposed by Modi et al. (2005) to estimate the loadability margin of the power system with static var compensator (SVC). Monte-Carlo simulation has been used by Arya et al. (2006) to evaluate probabilities of voltage collapse for various operating conditions of power system. An efficient technique for voltage stability assessment using a newly developed line voltage stability index that becomes half at a collapse point is presented by Arya et al. (2007) for power system. In an artificial neural network based approach is presented for voltage stability based contingency ranking of power system (Devaraj et al., 2007).

An optimal routing algorithm is presented by Shin et al. (2007) for minimizing power loss and at the same time maximizing the voltage stability index in radial power systems. Analytical approach to voltage collapse proximity determination is proposed for radial networks (Gubina and Strmcnik, 1997). A new voltage stability index is proposed for identifying the node in RDS, which is most sensitive to voltage collapse (Chakravorty and Das, 2001). A new methodology to financially analyze the investment in FACTS devices, to mitigate against voltage sags by incorporating financial aspects of the problem is reported by Ranjan and Das (2003). The methodology is based on the assessment of the total annual cost due to voltage sags on the entire network. Scenarios are evaluated with and without optimally placed FACTS devices. The capital investment cost and annual maintenance of devices over their life time is also considered.
A discounted cash flow analysis is used to understand the time dependent value of each mitigating solution. A bespoke Niching genetic algorithm (NGA) optimization is used by Jovica et al. (2010) to determine the location, size and type of different FACTS devices resulting in minimal network financial losses due to voltage sags. The full network annual financial losses due to voltage sags are used to guide the optimization process. NGA optimization is used as it identifies multiple feasible solutions that give more flexibility to the expert making the final investment decision. Generally, voltage stability can be classified into two subcategories: large-disturbance voltage stability and small-disturbance voltage stability. Large-disturbance voltage stability refers to the system’s ability to maintain steady voltages following large disturbances, such as system fault, loss of generation or circuit contingencies and the small-disturbance voltage stability refers to the system’s ability to maintain steady voltages when subjected to small perturbations such as incremental changes in system load. The small-disturbance voltage stability is mainly associated with reactive power imbalance. This imbalance mainly occurs on a local network or a specified bus in a system. If the reactive power on a local network has a shortage, the voltage in the network will decline and may be lower than the minimum threshold of normal voltage range. In the worst situation, it will lead to voltage collapse. If the reactive power on a local network exceeds the necessary level, voltage in the network will increase and may be higher than the maximum threshold of normal voltage range. Both situations need to be avoided in power systems operation to prevent disaster. Therefore, reactive power support must be provided. Zhu et al. (2010) focused on small-disturbance voltage stability and reports an application of a coordinated static var compensator (SVC) as additional control in reactive power (VAR) optimization problem and analyzes the impact on system loss minimization and voltage improvement.

A new static voltage stability index of a RDS is developed by Hamada et al. (2010), to faithfully evaluate the severity of the loading situation, thereby predicting for voltage instability at definite load value. The developed index includes different parameters which affect the steady-state voltage stability of distribution systems, therefore it gives accurate results. The maximum value of 1 of that index denotes the point, where the system reaches the point of collapse whereas a minimum value of 0 shows the state of no load. One of the major issues in improving power quality in distribution networks is, the mitigation of network voltage distortions for proper operation of industrial process, where there are sensitive and critical loads. The series active power filter seems an ideal device to improve power quality as it allows suppressing and isolating voltage based distortion such as voltage unbalance, sag, interruption and voltage harmonics.

The determination of voltage references for series active power filter based on a robust three phase digital locked loop system was analyzed by Mekria et al. (2010). Performances of two linear regulators: a polynomial controller based on a pole placement theory and a proportional integral derivative (PID) controller are studied and compared, in order to compensate different voltage perturbations in network and a novel hysteresis band voltage control, for a series active power filter is used to derive the switching signals and to improve the power quality. A novel control strategy is proposed by Ajaei et al. (2011) for independent control of the injected voltages in each phase of the DVR. The proposed control strategy effectively compensates the load voltage zero- and negative-sequence components, as well as the positive-sequence component. This enables the DVR to restore the load voltages during balanced and unbalanced sags, in a short time interval (5 ms), with zero steady-state error.

Electric Power Quality (PQ) can be defined as the capacity of an electric power system to supply electric energy of a load in an acceptable quality. Many problems can result from poor PQ, especially in today’s complex power systems, such as the false operation of modern control systems. Voltage sag is an important PQ problem because of sensitive loads growth. Worldwide experience has shown that short-circuit faults are the main origin of voltage sags and, therefore, there is a loss of voltage quality. Jafari et al. (2011) showed voltage sag compensation of point of common coupling (PCC) using a new structure of fault current limiter (FCL). The proposed structure prevents voltage sag and phase-angle jump of the substation PCC after fault occurrence. This structure has a simple control method. The Unified Power-Quality Conditioner (UPQC) is used to mitigate the current and voltage-related power-quality (PQ) problems simultaneously in power distribution systems. Among all of the PQ problems, voltage sag is a crucial problem in distribution systems.

A new methodology is proposed by Siva Kumar et al. (2011) to mitigate the unbalanced voltage sag with phase jumps by UPQC with minimum real power injection. Voltage-shift acceleration control for the anti-islanding of inverter based Distributed Generation (DGs) is proposed by Kim et al. (2011). Various studies shows that a lot of work has been done on voltage stability analysis of transmission systems, but hardly any work has been done on the voltage stability analysis of radial distribution networks (Gubina and Strmcnik, 1997; Chakravorty and Das, 2001; Shin et al., 2007).

In this study, a new voltage stability index for all the nodes is proposed for radial distribution networks. It is shown that the node, at which the value of voltage stability index is minimum, is more sensitive to voltage collapse.
Materials and methods

A simple load flow technique for solving radial distribution networks has been proposed by Ranjan and Das (2003). For the purpose of deriving the voltage stability index of radial distribution networks, this load flow technique will be explained in brief. In this section, a circuit model of a RDS is presented. While developing the circuit model it is assumed that the RDS is balanced and can be represented as a single line diagram as shown in Fig. 1. Line shunt capacitance at distribution voltage level is negligibly small. The mathematical model of radial distribution network can be easily derived from Fig. 2.

\[ Q(m2) = \text{sum of the reactive power loads of all the nodes beyond node } m2 \text{ plus reactive power load of the node } m2 \text{ itself plus the sum of the reactive power losses of all the branches beyond node } m2. \]

\[ |V(m2)| = \sqrt{(B(jj) - A(jj))} \]  

(3)

Where,
\[ A(jj) = P(m2) \times R(jj) + Q(m2) \]
\[ \times X(jj) - 0.5 \times |V(m1)|^2 \]

(4)

\[ B(jj) = \left\{ \frac{A^2(jj) - Z^2(jj)}{\sqrt{(P^2(m2) + Q^2(m2))}} \right\} \]

(5)

From Equation (3), it is seen that, a feasible load flow solution of radial distribution networks will exist if
\[ B(jj) - A(jj) \geq 0 \]

(6)

Substituting the value of \( B(jj) \) in the above Equation (6), we get
\[ \left\{ \frac{A^2(jj) - Z^2(jj)}{\sqrt{(P^2(m2) + Q^2(m2))}} \right\} - A(jj) \geq 0 \]

(7)

The above Equation (7) can be rewritten as
\[ \left[ A^2(jj) - Z^2(jj) \times \left( P^2(m2) + Q^2(m2) \right) \right]^{1/2} - A(jj) \geq 0 \]

(8)

Taking \( A(jj) \) common from Equation (8), we get
\[ A(jj) \times \left[ 1 - \left( \frac{Z(jj)}{A(jj)} \right)^2 \right] \times \left( P^2(m2) + Q^2(m2) \right)^{1/2} - A(jj) \geq 0 \]

(9)

Expanding Equation (9) using binomial expansion,
\[ A(jj) \times \left[ 1 + \frac{1}{2} \times \left( \frac{Z(jj)}{A(jj)} \right)^2 \right] \times \]
\[ \left( P^2(m2) + Q^2(m2) \right)^{1/2} + \frac{1}{2} \times \left( \frac{1}{2} - 1 \right) \times \left( \frac{Z(jj)}{A(jj)} \right)^4 \times \]
\[ \left( P^2(m2) + Q^2(m2) \right)^{1/2} + \ldots \ldots \ldots \]

\[ - A(jj) \geq 0 \]

©Youth Education and Research Trust (YERT)  Asha Gaikwad et al., 2013
Neglecting higher order terms, we get
\[
\frac{1}{2} \times \left[ \frac{Z^2(jj)}{A(jj)} \times (P^2(m2) + Q^2(m2)) \right] - \frac{1}{8} \times \left[ \frac{Z^2(jj)}{A(jj)} \times (P^2(m2) + Q^2(m2)) \right] \geq 0 \tag{11}
\]
Simplifying above Equation (11) we get,
\[
4 \times A^2(jj) - [Z^2(jj)] \geq 0
\]
Expanding the square term of Equation (13) we get,
\[
\frac{1}{2} \times \left[ \frac{Z^2(jj)}{A(jj)} \times (P^2(m2) + Q^2(m2)) \right] \geq 0
\tag{12}
\]
where
\[
l(jj) = R(jj) + X(jj)
\]
in Equation (1.12) we get,
\[
4 \times 0.5 \times V^2(ml) - (P(m2) \times R(jj)) + Q(m2) \times X(jj) - (P(m2) \times R(jj) + Q(m2) \times X(jj))^2 \geq 0
\tag{13}
\]
Expanding the square term of Equation (13) we get,
\[
\frac{1}{2} \times \left[ \frac{Z^2(jj)}{A(jj)} \times (P^2(m2) + Q^2(m2)) \right] \geq 0
\]
Expanding the square term of Equation (13) we get,
\[
4 \times 0.5 \times V^2(ml) - 2 \times 0.5 \times V^2(ml) \times (P(m2) \times R(jj) + Q(m2) \times X(jj)) + (P(m2) \times R(jj) + Q(m2) \times X(jj))^2 \geq 0
\tag{14}
\]
Simplifying the above Equation (14), we get
\[
V^4(ml) - 4 \times V^2(ml) \times (P(m2) \times R(jj) + Q(m2) \times X(jj)) + \left\{ (R^2(jj) + X^2(jj)) \times (P^2(m2) + Q^2(m2)) \right\} \geq 0
\tag{15}
\]
Let,
\[
SI(m2) = V^4(ml) - 4 \times V^2(ml) \times (P(m2) \times R(jj)) + (3 \times P^2(m2) \times R^2(jj)) + (3 \times Q^2(m2) \times X^2(jj)) + \left\{ (R^2(jj) + X^2(jj)) \times (P^2(m2) + Q^2(m2)) \right\} \geq 0
\tag{16}
\]
Where, $SI(m2)$ is the Voltage Stability Index (VSI) for receiving end node.

For stable operation of $m2$ the radial distribution network, $SI(m2) \geq 0$.

By using this voltage stability index, one can measure the level of stability of radial distribution networks and thereby appropriate action may be taken if the index indicates a poor level of stability. After the voltages at each node are calculated from load flow study, the voltage stability index $SI(m2)$ for all the receiving end nodes of RDS can be easily calculated using Equation (16). The node, at which the value of the stability index is minimum, is more sensitive to the voltage collapse.

**Results**

To demonstrate the effectiveness of the proposed method a 32-node RDS and 69-node RDS is considered. The data of 32-node RDS and 69-node RDS are taken from Chakravorty and Das (2001) and Ranjan and Das (2003). The real and reactive power loads of node i are given as:
\[
PL(i) = f \times PL_o(i) \tag{17}
\]
\[
QL(i) = f \times QL_o(i) \tag{18}
\]

In Equations (17) and (18), $f$ is a scaling factor and $f$ is varied from zero to a critical value at which voltage collapse takes place, i.e. loads are gradually increased at every node. After carrying out load flow studies for 32-node RDS, it was found that for 32-node RDS minimum voltage occurred at 18th node. Also it was found when the load of 32-node system was increased gradually, the minimum value of voltage stability index is occurring at node 18. Therefore, it can be concluded that node 18 is more sensitive to voltage collapse. After carrying out load flow studies for 69-node RDS, it was found that for 69-node RDS minimum voltage occurred at 65th node. Also it was found when the load of 69-node system was increased gradually, the minimum value of voltage stability index is occurring at node 65. Therefore, it can be concluded that node 65 is more sensitive to voltage collapse. Table 1 shows that the maximum total real power load and total reactive power load that can be supplied by 32-node RDS without voltage collapse at any node is 12,874.1 kW and 8154.390 kVar respectively. Table 2 shows that the maximum total real power load and total reactive power load that can be supplied by 69-node RDS without voltage collapse at any node is 13,666.56kW and 8891.365 kVar respectively. From Table 3 it can be seen that, by both methods, for 69-node RDS, the node which is more sensitive to voltage collapse is 65th node. Thus, it can be concluded that the voltage stability index derived in this study is correct.
Table 1. Critical loading conditions for constant power load of 32-node System.

<table>
<thead>
<tr>
<th>Iteration no.</th>
<th>Voltage at node 18 (V_{18}) (kV)</th>
<th>Voltage Stability Index (S_I(18))</th>
<th>(P_L(18)) (kW)</th>
<th>(Q_L(18)) (kVAR)</th>
<th>(P_{(m2)}) (kW)</th>
<th>(Q_{(m2)}) (kVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.44</td>
<td>17094.23</td>
<td>90</td>
<td>40</td>
<td>3913.6999</td>
<td>2436.76</td>
</tr>
<tr>
<td>2</td>
<td>11.44</td>
<td>17094.23</td>
<td>90</td>
<td>40</td>
<td>3913.69</td>
<td>2436.76</td>
</tr>
<tr>
<td>3</td>
<td>11.41</td>
<td>16933.92</td>
<td>91.8</td>
<td>40.8</td>
<td>3996.73</td>
<td>2488.78</td>
</tr>
<tr>
<td>4</td>
<td>11.36</td>
<td>16608.32322</td>
<td>95.47</td>
<td>42.43</td>
<td>4166.80</td>
<td>2595.37</td>
</tr>
<tr>
<td>5</td>
<td>11.27</td>
<td>16104.23</td>
<td>101.20</td>
<td>44.97</td>
<td>4434.02</td>
<td>2762.97</td>
</tr>
<tr>
<td>6</td>
<td>11.14</td>
<td>15399.79</td>
<td>109.29</td>
<td>48.57</td>
<td>4815.77</td>
<td>3002.67</td>
</tr>
<tr>
<td>7</td>
<td>10.97</td>
<td>14463.73</td>
<td>120.22</td>
<td>53.43</td>
<td>5339.18</td>
<td>3331.84</td>
</tr>
<tr>
<td>8</td>
<td>10.74</td>
<td>13254.54</td>
<td>134.65</td>
<td>59.84</td>
<td>6045.43</td>
<td>3776.99</td>
</tr>
<tr>
<td>9</td>
<td>10.41</td>
<td>11720.14</td>
<td>153.50</td>
<td>68.22</td>
<td>6997.77</td>
<td>4379.14</td>
</tr>
<tr>
<td>10</td>
<td>9.96</td>
<td>9799.02</td>
<td>178.06</td>
<td>79.14</td>
<td>8297.84</td>
<td>5204.95</td>
</tr>
<tr>
<td>11</td>
<td>9.30</td>
<td>7424.76</td>
<td>210.11</td>
<td>93.38</td>
<td>10125.27</td>
<td>6373.97</td>
</tr>
<tr>
<td>12</td>
<td>8.23</td>
<td>4530.44</td>
<td>252.14</td>
<td>112.06</td>
<td>12874.1</td>
<td>8154.39</td>
</tr>
</tbody>
</table>

No solution exists

Table 2. Critical loading conditions for constant power load of 69-node System.

<table>
<thead>
<tr>
<th>Iteration no.</th>
<th>Voltage at node 18 (V_{65}) (kV)</th>
<th>Voltage Stability Index (S_I(65))</th>
<th>(P_L(65)) (kW)</th>
<th>(Q_L(65)) (kVAR)</th>
<th>(P_{(m2)}) (kW)</th>
<th>(Q_{(m2)}) (kVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.51</td>
<td>17548.34</td>
<td>59</td>
<td>42</td>
<td>4016.76</td>
<td>2796.02</td>
</tr>
<tr>
<td>2</td>
<td>11.51</td>
<td>17548.34</td>
<td>59</td>
<td>42</td>
<td>4016.76</td>
<td>2796.02</td>
</tr>
<tr>
<td>3</td>
<td>11.48</td>
<td>17393.94</td>
<td>60.18</td>
<td>42.84</td>
<td>4102.56</td>
<td>2854.40</td>
</tr>
<tr>
<td>4</td>
<td>11.43</td>
<td>17079.98</td>
<td>62.58</td>
<td>44.55</td>
<td>4278.39</td>
<td>2973.84</td>
</tr>
<tr>
<td>5</td>
<td>11.35</td>
<td>16592.94</td>
<td>66.34</td>
<td>47.22</td>
<td>4554.91</td>
<td>3161.15</td>
</tr>
<tr>
<td>6</td>
<td>11.23</td>
<td>15910.26</td>
<td>71.64</td>
<td>51.00</td>
<td>4950.54</td>
<td>3428.03</td>
</tr>
<tr>
<td>7</td>
<td>11.06</td>
<td>14999.13</td>
<td>78.81</td>
<td>56.10</td>
<td>5494.16</td>
<td>3792.53</td>
</tr>
<tr>
<td>8</td>
<td>10.84</td>
<td>13814.74</td>
<td>88.27</td>
<td>62.83</td>
<td>6230.08</td>
<td>4281.78</td>
</tr>
<tr>
<td>9</td>
<td>10.53</td>
<td>12298.04</td>
<td>100.63</td>
<td>71.63</td>
<td>7227.56</td>
<td>4936.81</td>
</tr>
<tr>
<td>10</td>
<td>10.09</td>
<td>10372.57</td>
<td>116.73</td>
<td>83.09</td>
<td>8601.15</td>
<td>5822.55</td>
</tr>
<tr>
<td>11</td>
<td>9.44</td>
<td>7937.00</td>
<td>137.74</td>
<td>98.05</td>
<td>10565.34</td>
<td>7052.83</td>
</tr>
<tr>
<td>12</td>
<td>8.33</td>
<td>4815.32</td>
<td>165.29</td>
<td>117.66</td>
<td>13666.56</td>
<td>8891.36</td>
</tr>
</tbody>
</table>

No solution exists

Table 3. Comparison of results obtained for 69-node system.

<table>
<thead>
<tr>
<th>Method used</th>
<th>Substation Voltage (p.u)</th>
<th>Critical loading conditions</th>
<th>Node sensitive to voltage collapse</th>
<th>Voltage Stability Index</th>
<th>(S_I(65)) (p.u)</th>
<th>(P(m2)) (MW)</th>
<th>(Q(m2)) (MVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method</td>
<td>1.0</td>
<td>13.66</td>
<td>8.89</td>
<td>65</td>
<td>0.048</td>
<td>6502.84</td>
<td></td>
</tr>
<tr>
<td>Chakravorty and Das (2001)</td>
<td>1.0</td>
<td>12.21</td>
<td>8.65</td>
<td>65</td>
<td>0.049</td>
<td>6502.84</td>
<td></td>
</tr>
</tbody>
</table>

©Youth Education and Research Trust (YERT)  Asha Gaikwad et al., 2013
Figure 3 and 4 shows the plots of $V(18)$ vs Total Real Power Load (TPL) and $V(18)$ vs Total Reactive Power Load (TQL) for different substation voltages. Point A indicates the critical loading point beyond which a small increment of loading causes the voltage collapse. Figure 5 and 6 shows the plots of $S_l(18)$ vs TPL and $S_l(18)$ vs TQL. Point A indicates the critical loading point beyond which a small increment of loading causes the voltage collapse.

Figure 7 and 8 shows the plots of $V(65)$ vs TPL and $V(65)$ vs TQL for different substation voltages. Point A indicates the critical loading point beyond which a small increment of loading causes the voltage collapse. Figure 9 and 10 shows the plots of $S_l(65)$ vs TPL and $S_l(65)$ vs TQL. Point A indicates the critical loading point beyond which a small increment of loading causes the voltage collapse.
Conclusion
A new voltage stability index has been proposed for radial distribution networks. Using this voltage stability index, it is possible to compute the stability index value at every node and the node at which the value of the voltage stability index is minimum is, most sensitive to voltage collapse. The effectiveness of the proposed technique has been demonstrated through a 32-node and 69-node radial distribution network and the results obtained are compared with the existing method. It is found that with both the existing voltage stability index (Chakravorty and Das, 2001) and with the proposed voltage stability index for 69-node system, the node which is more sensitive to voltage collapse is node-65.

Acknowledgements
The authors are thankful to Dr. Rakesh Ranjan and Dr. L.D. Arya for their guidance provided at every step to carry out this research work.

References