Reliability optimization of integrated reliability model using dynamic programming and failure modes effects and criticality analysis

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Abstract

The calculation of number of components, component’s reliability, stage reliability, and the system reliability represents an integrated reliability model (IRM) according to Chern and Jan (1986) and Kuo and Rajendra Prasad (2000). So far in literature, the IRM models are optimized only using cost constraint. The classic approach of this work is to optimize a class of IRM for redundant systems with volume and weight as the other constraints. The effort has been made for the given mathematical function to establish the results by using Lagrangean multiplier method, but these results in real values of number of components required. Rounding off to the next integer value, results in major variation in all the constraints. Dynamic Programming results in very little variation in all the constraints. The novel approach of failure modes effects and criticality analysis (FMECA) is that it finds and corrects the causes and also compliments the results obtained through the dynamic programming.

Keywords: Integrated reliability model, cost constraint, redundant systems, Lagrangen multiplier method.

Introduction

Reliability optimization has allowed a great deal of attention from researchers for almost six decades due to its serious importance in various kinds systems (Chern and Jan, 1986; Kuo and Rajendra Prasad, 2000). For many engineering products, the quality of operation and maintenance also influences reliability. There is no fundamental limit to the extent to which failures can be prevented. We can design and build for ever-increasing reliability. It is easy to manufacture cost effective systems that perform better by blending reliability concepts in all phases of the product life cycle from proposal to manufacture. The reliability of a system can be maximized subject to the resource constraint to determine the optimum number of redundant components for each stage when the reliability of each component is known. In other situation, the reliability of the system can be maximized subject to the resource constraint to determine the reliabilities of the components in the system when the number of redundant units in each stage is known. Literature survey reveals the available techniques to solve the problems in these two situations. Optimization is associated almost exclusively with the use of mathematical models and analyzes decision problems (Gorden, 1957). Usually these mathematical and stochastic models are tailored to fit into specific real life problems. It is difficult to conceive a model that reflects the reality as close as possible and at the same time simple for analysis. Therefore, different models each depicting one or more problem situations are developed. A substantial body of literature has been brought on reliability models for the past two decades.

Statement of the problem

The problem considers the component reliabilities and the number of components in each stage are unknown for the given constraints to maximize the system reliability. An attempt was made in this study to negotiate the impact of weight and volume as constraints in optimizing the redundant systems under consideration for the selected mathematical function. Though cost has direct relation in maximizing system reliability, the indirect impact of weight and volume as constraints in optimizing the reliability of a redundant system presents a novel beginning in this area of research (Lakshminarayana et al., 2008). The series-parallel systems are considered with cost, weight and volume as constraints to maximize the reliability of a redundant system as its objective function.

Assumptions of the model

1. All the components in each stage are assumed to be identical.
2. The failure of one component does not affect the performance of the other components in the system.
3. A component is either in working condition or non-working condition.

Mathematical model

The objective function and the constraints of the model are

\[
\text{Max } R_s = \prod_{j=1}^{n} R_j = \prod_{j=1}^{n} [1 - (1 - r_j)^{x_j}] 
\]

Where:

- \(R_s\) is the system reliability.
- \(R_j\) is the stage reliability.
- \(r_j\) is the component reliability.
- \(x_j\) is the number of components in each stage.
- \(n\) is the number of stages.

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- \(x_j\) is the number of components in each stage.
- \(n\) is the number of stages.
Subjected to
\[
\begin{align*}
\sum_{j=1}^{n} c_j \cdot x_j & \leq C_0 \quad \ldots \; 1 \\
\sum_{j=1}^{n} w_j \cdot x_j & \leq W_0 \quad \ldots \; 2 \\
\sum_{j=1}^{n} v_j \cdot x_j & \leq V_0 \quad \ldots \; 3
\end{align*}
\]

Non-negativity restriction \(x_j\) is an integer and \(r_j, R_j > 0\)

Where
\( R = \) System reliability
\( R_j = \) Stage reliability \(0 < R_j < 1\)
\( r_j = \) Reliability of each component in stage \(j\), \(0 < r_j < 1\)
\( x_j = \) No. of components in stage \(j\)
\( c_j = \) Cost coefficient of each component in stage \(j\)
\( w_j = \) Weight coefficient of each component in stage \(j\)
\( v_j = \) Volume coefficient of each component in stage \(j\)
\( C_0 = \) Maximum allowable system cost
\( W_0 = \) Maximum allowable system weight
\( V_0 = \) Maximum allowable system volume

System Reliability for the given cost function is

\[ R_s = \prod_{j=1}^{n} R_j \]

Cost coefficient of each component in stage \(j\) is derived from the following relationship between cost and reliability

\[ r_j = \left( \frac{c_j}{b_j} \right)^{1/d_j} \]

\( c_j = b_j \cdot r_j^{d_j} \quad \ldots \; \text{Cost constraint} \]

Similarly using the same relationship weight and volume constraints are

\[ w_j = p_j \cdot r_j^{q_j} \quad \ldots \; \text{Weight constraint} \]

\[ v_j = k_j \cdot r_j \quad \ldots \; \text{Volume constraint} \]

The stationery point can be obtained by differentiating the Lagrangean function with respect to \(R_j\), \(r_j\), \(\lambda_1\), \(\lambda_2\), and \(\lambda_3\) and the problem can be rewritten after simplification

\[ \frac{\partial F}{\partial \lambda_1} = \sum_{j=1}^{n} \left( b_j \cdot r_j^{q_j} \ln(1-R_j) \right) - C_0 = 0 \]

\[ \frac{\partial F}{\partial \lambda_2} = \sum_{j=1}^{n} \left( p_j \cdot r_j^{q_j} \ln(1-R_j) \right) - W_0 = 0 \]

\[ \frac{\partial F}{\partial \lambda_3} = \sum_{j=1}^{n} \left( k_j \cdot r_j \ln(1-R_j) \right) - V_0 = 0 \]

Case

Consider the case of a Mechanical system with three stages for which the component Reliability is given by the equation:

\[ r_j = \left[ \frac{c_j}{b_j} \right]^{\frac{1}{d_j}} \]

To determine the optimum component reliability \(r_j\), stage reliability \(R_j\), number of components in each stage \(x_j\), and the system reliability \(R_s\) not to exceed the system cost: Rs. 300, weight of the system: 400 Kgs and volume of the system: 600 cm\(^3\). The component reliabilities, stage reliabilities, number of components in each stage and the system reliability are determined by solving the above mathematical function using MATLAB.

Cost, weight and volume as constraints

Reliability design relating to cost, weight and volume is shown in Table 1, 2 and 3.

The stationery point can be obtained by differentiating the Lagrangean function with respect to \(R_j\), \(r_j\), \(x_j\), \(c_j\), \(c_x\), and \(x_j\) rounding off

By contemplating the values of \(x_j\) to be integers (by rounding off the value of \(x_j\) to the nearest integer) and the relevant results relating to cost, weight and volume are presented in the tables 4, 5 and 6, further calculated the variation due to cost, weight, volume and system reliability.

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System reliability: Rs. 0.937
Variation in total cost = 29.33%
Variation in total weight = 32.25%
Variation in total volume = 26.60%
Variation in system reliability = 5.56%.

**Dynamic programming**

To optimize the design by using dynamic programming the same case problem discussed in the preceding chapter has been considered by taking the values of component reliabilities ($r_i$), the number of components in each stage ($x_i$), stage reliabilities ($R_i$) and the system reliability ($R_s$) as inputs. This approach is particularly useful in optimizing the design with the values of $x_i$'s to be integers, which are highly appreciated for practical implementation to real life problems. The necessary program is developed in C language with inputs taking from the Lagrangean method. The number of components, which was taken as a real number has been changed to an integer. The output has come in two stages with corresponding Stage Reliability. The results are shown in Table 7 to 12.

<table>
<thead>
<tr>
<th>Stage</th>
<th>$r_i$</th>
<th>$R_i$</th>
<th>$x_i$</th>
<th>$c_i$</th>
<th>$c_i\cdot x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.9471</td>
<td>0.9701</td>
<td>2</td>
<td>54</td>
<td>108</td>
</tr>
<tr>
<td>02</td>
<td>0.9608</td>
<td>0.9402</td>
<td>2</td>
<td>44</td>
<td>088</td>
</tr>
<tr>
<td>03</td>
<td>0.9882</td>
<td>0.9863</td>
<td>4</td>
<td>48</td>
<td>192</td>
</tr>
</tbody>
</table>

**Table 4. Variation due to cost.**

<table>
<thead>
<tr>
<th>Stage</th>
<th>$r_i$</th>
<th>$R_i$</th>
<th>$x_i$</th>
<th>$c_i$</th>
<th>$c_i\cdot x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.9471</td>
<td>0.9701</td>
<td>2</td>
<td>87.91</td>
<td>176</td>
</tr>
<tr>
<td>02</td>
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<td>0.9402</td>
<td>2</td>
<td>62.09</td>
<td>124</td>
</tr>
<tr>
<td>03</td>
<td>0.9882</td>
<td>0.9863</td>
<td>4</td>
<td>57.22</td>
<td>229</td>
</tr>
</tbody>
</table>

**Table 5. Variation due to weight.**

<table>
<thead>
<tr>
<th>Stage</th>
<th>$r_i$</th>
<th>$R_i$</th>
<th>$x_i$</th>
<th>$c_i$</th>
<th>$c_i\cdot x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.9471</td>
<td>0.9701</td>
<td>2</td>
<td>89.70</td>
<td>179</td>
</tr>
<tr>
<td>02</td>
<td>0.9608</td>
<td>0.9402</td>
<td>2</td>
<td>88.70</td>
<td>177</td>
</tr>
<tr>
<td>03</td>
<td>0.9882</td>
<td>0.9863</td>
<td>4</td>
<td>101.09</td>
<td>404</td>
</tr>
</tbody>
</table>

**Table 6. Variation due to volume.**

<table>
<thead>
<tr>
<th>No. of components-$x_i$</th>
<th>Stage reliability-$R_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.8271</td>
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<tr>
<td>02</td>
<td>0.9701</td>
</tr>
<tr>
<td>03</td>
<td>0.9948</td>
</tr>
<tr>
<td>04</td>
<td>0.9991</td>
</tr>
<tr>
<td>05</td>
<td>0.9998</td>
</tr>
<tr>
<td>06</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

**Table 7. Dynamic programming-Stage 1.**

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>Stage reliability-$R_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>02</td>
<td>0.9099</td>
</tr>
<tr>
<td>03</td>
<td>0.9436 0.9321</td>
</tr>
<tr>
<td>04</td>
<td>0.9468 0.9666 0.9353</td>
</tr>
<tr>
<td>05</td>
<td>0.8270 0.9698 0.9912 0.9394</td>
</tr>
</tbody>
</table>

**Table 8. Dynamic programming-Stage 2.**

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>Stage reliability-$R_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>03</td>
<td>0.9881 0.9428 0.9776 0.9963 0.9320 0.9099</td>
</tr>
</tbody>
</table>

**Table 9. Dynamic programming-Stage 3.**

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>Stage reliability-$R_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.9692</td>
</tr>
<tr>
<td>4</td>
<td>0.9211 0.8580</td>
</tr>
<tr>
<td>5</td>
<td>0.9552 0.8789 0.8975</td>
</tr>
<tr>
<td>6</td>
<td>0.9795 0.9114 0.9193 0.9070</td>
</tr>
<tr>
<td>7</td>
<td>0.9881 0.9346 0.9534 0.9290 0.9099</td>
</tr>
<tr>
<td>8</td>
<td>0.9881 0.9428 0.9776 0.9963 0.9320 0.9099</td>
</tr>
</tbody>
</table>

**Reliability design-cost, weight and volume**

From the dynamic programming tables the maximum system reliability is 0.9167 with a total cost of Rs. 287.9 and the corresponding optimal values are as shown in Table 10. From the dynamic programming tables the maximum system reliability is 0.9167 with a total weight of 383.75 kg and the corresponding optimal values are as shown in Table 11. From the dynamic programming tables the maximum system reliability is 0.9167 with a total volume of 570 cc and the corresponding optimal values are as shown in Table 12.

<table>
<thead>
<tr>
<th>Stage</th>
<th>$r_i$</th>
<th>$R_i$</th>
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<td>48</td>
<td>192</td>
</tr>
</tbody>
</table>

**Table 9. Dynamic programming-Stage 3.**

<table>
<thead>
<tr>
<th>Stage</th>
<th>$r_i$</th>
<th>$R_i$</th>
<th>$x_i$</th>
<th>$c_i$</th>
<th>$c_i\cdot x_i$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.9701</td>
<td>1</td>
<td>54</td>
<td>54</td>
</tr>
<tr>
<td>02</td>
<td>0.9608</td>
<td>0.9666</td>
<td>2</td>
<td>44</td>
<td>89</td>
</tr>
<tr>
<td>03</td>
<td>0.9882</td>
<td>0.9776</td>
<td>3</td>
<td>48</td>
<td>144.9</td>
</tr>
</tbody>
</table>

**Table 10. Reliability design-cost.**

<table>
<thead>
<tr>
<th>Stage</th>
<th>$r_i$</th>
<th>$R_i$</th>
<th>$x_i$</th>
<th>$c_i$</th>
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</thead>
<tbody>
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<tr>
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<td>0.9608</td>
<td>0.9666</td>
<td>2</td>
<td>62.09</td>
<td>124.18</td>
</tr>
<tr>
<td>03</td>
<td>0.9882</td>
<td>0.9776</td>
<td>3</td>
<td>57.22</td>
<td>171.66</td>
</tr>
</tbody>
</table>

**Table 11. Reliability design-weight.**

<table>
<thead>
<tr>
<th>Stage</th>
<th>$r_i$</th>
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<th>$x_i$</th>
<th>$c_i$</th>
<th>$c_i\cdot x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.9471</td>
<td>0.9701</td>
<td>1</td>
<td>89.70</td>
<td>89.70</td>
</tr>
<tr>
<td>02</td>
<td>0.9608</td>
<td>0.9666</td>
<td>2</td>
<td>88.70</td>
<td>177.40</td>
</tr>
<tr>
<td>03</td>
<td>0.9882</td>
<td>0.9776</td>
<td>3</td>
<td>101.09</td>
<td>303.27</td>
</tr>
</tbody>
</table>

**Table 12. Reliability design-volume.**

System reliability = 0.9167
Variation in total cost = 4.033%
Variation in total weight = 4.063%
Variation in total volume = 4.938%
Variation in system reliability = 7.581%.

**Failure modes effects and criticality analysis**

Failure modes effects and criticality analysis supports safety engineering effects in analysis such as the fault tree analysis (Smith, 1985; Venkataraman, 2007). The failure modes with their assigned criticality would be seen as basic events. As part of the maintainability analysis, failure modes effects and criticality analysis is the importance that detection and isolation is accurately reflected in the overall mean time to repair calculations.
This analysis would support the design engineering effort to ensure that programme design requirements are taken care of, which could be in the support of requirements like as no single points of failure. The failure modes effects and criticality analysis and criticality analysis can be implemented as a functional analysis and or physical analysis. Functional analysis approach would be taken earlier in a design process. With improved definition of the design and as more details are firm up then this will permit a physical analysis to be implemented. The analysis effectively provides a contribution to final system configuration, with regards to reliability performance characteristics, during the actual design phase. It is to be noted that reliability centered maintenance treats the symptom while this analysis finds and corrects the cause. That means to say that the purpose of the analysis is to uncover the underlying reasons (the root causes) why an event is occurring so that the necessary steps can be taken to eliminate the event in its entirety by analyzing the modes (Kelly and Harris, 1978; Smith, 1985; Leitch, 1995; Venkataraman, 2007).

Criticality analysis/criticality matrix
Once the failure mode is identified in the failure modes effects analysis, then the purpose of criticality analysis is to rank each failures mode according to the severity and the probability of occurrence of the same. This results in a criticality matrix. The next step is to divide the criticality scale into a number of sections according to the probability of occurrence to represent Z axis of the criticality matrix.

Level 1, Frequent: The probability which is greater than or equal to 0.2 (≥20%) of the overall system probability of failure.
Level 2, Moderate: The probability which is between 0.1 and 0.2 (10 to 20%) of the overall system probability of failure.
Level 3, Occasional: The probability between 0.01 and 0.1 (1 to 10%) of the overall system probability of failure.
Level 4, Remote: The probability between 0.001 and 0.01 (0.1 to 1%) of the overall system probability of failure.
Level 5, Very unlikely: The probability which is less than 0.001 (<0.1%) of the overall system probability of failure.

The next step is to calculate the severity classification as follows:
Category I, Catastrophic: A failure that may cause death or total system loss (that is, aircraft, vehicle, missile, ship, Train etc).
Category II, Critical: A failure that may cause severe injury, major property damage, or major system damage, which results in considerable loss.
Category III, Marginal: A failure that may cause minor injury, minor property damage or minor system damage, which results in a delay or degradation.

Category IV, Minor: A failure which is not serious enough to cause injury, property damage or system damage, which results in unscheduled repair or maintenance.

The criticality matrix is constructed as shown in Figure 1, x-axis representing the severity, y-axis representing the criticality number, z-axis representing the level.

![Criticality matrix](image)

Fig. 1. Criticality matrix.

Conclusion
Till date not much of work is reported on integrated reliability models for redundant systems. All most all the models that are reported primarily considered cost as the basic constraint. In this scenario, the authors proposed a class of integrated reliability models for redundant systems with multiple constraints as a novel beginning in the mentioned area of research and initiated the optimizing the system reliability for the said model under two different approaches and the results reported in the work is highly useful for the reliability/design engineers for successful implementation which helps to produce highly reliable and quality goods and the models are established for the series-parallel reliable configuration systems. This model can also be further investigated for different mathematical functions of interest and also can be applied for parallel-series configuration systems, where the application of these models for such systems will be feasible only when the cost of the system is very low. The authors are of the opinion that the stated problem can be investigated under the scope of study. The Lagrangean approach has given the reliability of three stage system is 0.8521 where the number of components are real, which has been increased to 0.937 by just rounding off the number of components to the nearest integer, since the number of components cannot be real numbers. The system reliability has gone to 0.9167 when calculated scientifically by using dynamic programming which has taken care of cost, weight, volume and the number of components is integers. For the system reliability of 0.9167, this indicated that the percentage failure was 8.33%. This takes the position of level-3 (z-axis) in the criticality matrix, which is very close to the safe zone. The novel approach of FMECA is that it finds and corrects the cause and also reinforces the results obtained through the dynamic programming.
References